Kumar's, Zipf 's and Other Laws: How to Structure a Large-Scale Wireless Network?

Mischa Dohler, Thomas Watteyne, Fabrice Valois, Jia-Liang Lu

Noname manuscript No.
(will be inserted by the editor)

Kumar's, Zipf's and Other Laws:

How to Structure a Large-Scale Wireless Network?

Mischa Dohler · Thomas Watteyne
Fabrice Valois · Jia-Liang Lu

Received: 3 September 2007 / Accepted: 5 February 2008

Abstract Networks with a very large number of nodes are known to suffer from scalability problems, influencing throughput, delay and other quality of service (QoS) parameters. Mainly applicable to wireless sensor networks, this paper extends the work of [1] and aims to give some fundamental indications on a scalable and optimum (or near-optimum) structuring approach for large-scale wireless networks. Scalability and optimality will be defined w.r.t. various performance criteria, an example of which is the throughput per node in the network. Various laws known from different domains will be invoked to quantify the performance of a given topology; most notably, we will make use of the well-known Kumar's law, as well as less known Zipf's and other scaling laws. Optimum network structures are derived and discussed for a plethora of different scenarios, facilitating knowledgeable design guidelines for these types of networks.

Keywords scalability · large-scale networks · throughput

This work is partially funded by the French ARESA RNRT Project ANR-05-RNRT-01703.

M. Dohler
CTTC, Parc Mediterrani, Av. Canal Olimpic, 08860 Castelldefels, Barcelona, Spain
Tel.: +34-93-645 2900
Fax: +34-93-645 2901
E-mail: first.lastname@cttc.es

T. Watteyne
France Télécom R&D, Meylan Cedex, F-38243, France
Tel.: +33-4-7676 2664
Fax: +33-4-7676 4450
E-mail: first.lastname@orange-ftgroup.com

F. Valois & J.-L. Lu
ARES INRIA / CITI, INSA-Lyon, F-69621, France
Tel.: +33-4-7243 6418
Fax: +33-4-7243 6227
E-mail: first.lastname@insa-lyon.fr

1 Introduction

Mobile operators offer today a variety of services to its clientele, including mobile and fixed telephony, wired and wireless Internet, as well as integrated home and business solutions. They own large scale wired and wireless networks, with the latter traditionally being composed of cellular networks and lately also of wireless sensor networks (WSNs). Operators' networks are composed of several million of nodes and enjoy planning and optimization prior to roll-out. WSNs are expected to be composed of several tens of thousands of nodes and generally do not enjoy planned roll-outs.

Through **cellular systems**, mobile operators have already been offering 2G, 3G and 3.5G wireless voice and date services for some years. Whilst subscriber numbers were low at the beginning, these have risen dramatically over the past years and hence having triggered the need to continuously augment the capacity of the cellular network. The invoked solution consists of introducing a hierarchical communication structure in form of cells, where several users are connected to a base station (BS), several of these BSs are then connected to a network controller, and the network controllers are then meshed by means of a backbone.

Whilst this facilitated scalability, such a solution is clearly expensive; for instance, a 3G Node B may cost several hundred thousand Euros. The question hence arises whether the approach taken is optimum or whether another solution would have been more appropriate. Whilst the answer depends on many factors and the limited scope of the paper prohibits all of these to be taken into account, we aim to give some indicative answers on the optimality of the hierarchical approach.

As for **wireless sensor networks**, respective companies as well as operators hope to offer more complete services by creating and facilitating ambient environments, which interface with incumbent and emerging services in industrial and home businesses. For this reason, these companies have strong R&D activities in the area of WSNs − exemplified by the national ARESA research project [2].

Sensor networks have been researched and deployed for decades already; their wireless extension, however, has witnessed a tremendous upsurge in recent years. This is mainly attributed to the unprecedented operating conditions of WSNs, i.e. a potentially enormous amount of sensor nodes reliably operating under stringent energy constraints. WSNs allow for an untethered sensing of the environment. It is anticipated that within a few years, sensors will be deployed in a variety of scenarios, ranging from environmental monitoring to health care, from the public to the private sector, etc; a deployment example is shown in Figure 1 [3]. They will be battery-driven and deployed in great numbers at once in an ad hoc fashion, requiring communication protocols and embedded system components to run in an utmost energy efficient manner.

Prior to large-scale deployment, however, a gamut of problems has still to be solved which relates to various issues, such as the design of suitable software and hardware architectures, development of communication and organization protocols, validation and first steps of prototyping, until the actual commercialization. In contrast to known and well understood systems, however, a WSNs bear some fundamental design differences, i.e. [4]:

- **Number of Nodes:** The number of nodes involved is very large, where current rollout examples include a few thousands; however, roll-out expectations are in the range of a few hundred thousand nodes communicating simultaneously. This is also

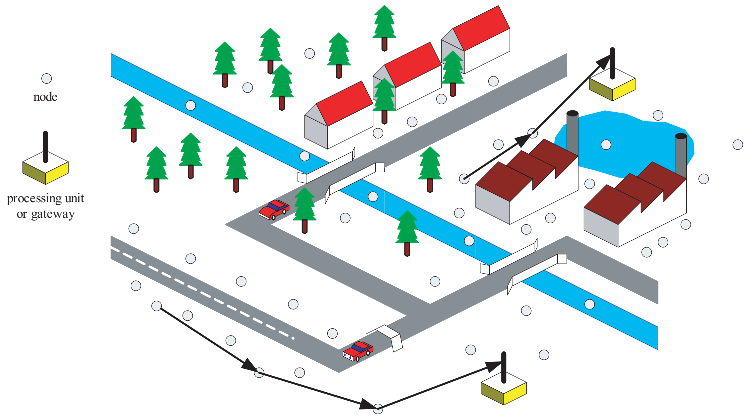

Fig. 1 Environmental monitoring by randomly distributed WSN nodes reporting to one or several processing units/gateways.

atypical to any wireless system today, and hence poses new technological as well as social and environmental challenges.
– **Energy:** WSNs are nowadays battery powered and, because changing batteries in a few thousand nodes on a regular basis is clearly impractical, they are required to have a long lifetime and are hence considered to be highly constrained in energy. This is in contrast to other systems, where nodes are usually either powered by the mains or easily rechargeable on a regular basis.
– **Applications:** The gamut of applications is vast, hence requiring very different solutions to be developed for different applications. This problem is further enhanced due to the stringent energy constraints, requiring subtle solutions to be developed for different requirements.

This means that, unlike incumbent systems, wireless sensor networks need to be [4]:

– highly scalable (protocols ought to work at arbitrary number of nodes);
– highly energy efficient (at all layers and functionalities); and
– highly application tailored (efficient for given task).

Of prime concern among industrialists, however, is currently the WSNs' scalability [5]. The aim of this paper is hence to give some insights into the scalability issue of these and other networks with a very large number of nodes. To this end, the paper is structured as follows. In Section 2, we dwell on the definition of 'scalability' and 'optimum'. In Section 3, we will discuss the role and importance of various scaling laws. Section 4 is dedicated to the application of these scaling laws to different communication structures. Finally, conclusions are drawn in Section 5.

2 Definitions

Before embarking onto the quantification of optimum scalable architectures and various scaling laws, we shall subsequently define 'scalability' and 'optimality'.

2.1 Scalability

An algorithm or architecture is said to be scalable if it can handle increasingly bigger and complex problems. Whilst such basic notion is intuitive, the term 'scalability' has so far evaded a generally-accepted definition. To this end, Hill [6] claims that the "use of the term adds more to marketing potential than technical insight". He concludes that no rigorous definition is applicable to scalability and he challenges "the technical community to either rigorously define scalability or stop using it to describe systems." To this end, the milestone paper [8] aimed at quantifying scalability of ad hoc routing protocols in a rigorous way. To attempt a more general definition that is applicable to large-scale wireless systems, we will modify the approach taken by [6]−[8]. We first introduce

$$\eta_{12} = \frac{\mathcal{F}_{\mathcal{A}}(N_2, S_2)/N_2}{\mathcal{F}_{\mathcal{A}}(N_1, S_1)/N_1} \tag{1}$$

to be the relative efficiency between two systems obeying the same type of architecture $\mathcal{A}$, consisting of N_1 and N_2 nodes, respectively, tackling some problems of size S_1 and S_2, respectively, and being gauged by some 'positive' average network-wide attribute $\mathcal{F}$. This 'positive' attribute could, e.g., be the total average network throughput, the inverse of the average end-to-end delay, etc; positivity here refers to the fact that increasing the value of the attribute improves performance. The problem size is determined by the 'problem' the system aims to solve and is related to the attribute; e.g., the problem of a network may be to deliver a given amount of data from every node (cellular system), or to measure and deliver a fixed set of measurements (data aggregating WSN), or simply to deliver just one measurement/bit (alert triggered WSN), etc.

To facilitate a definition of scalability, we assume the following:

- The difference between the number of nodes in the two systems approaches infinity, i.e., with $N_1 = N$ and $N_2 = N + \Delta$, we require $\Delta \to \infty$. The requirement on Δ approaching infinity stems from the fact that the below-given ratio (2) can often only be calculated in closed form under this assumption.
- N is sufficiently large such that the attribute $\mathcal{F}$ holds with sufficiently high probability. The requirement on N being sufficiently large stems from the fact that many network-wide attributes, such as average throughput and delay, can only be quantified if the number of involved nodes is sufficiently large (often even infinite).
- The problem size of the larger system does not decrease, i.e. $S_2 \geq S_1$. This means that the system with a larger number of nodes is not required to perform a more trivial task.

With these assumptions, we now define an architecture $\mathcal{A}$ to be scalable w.r.t. attribute $\mathcal{F}$ if

$$\eta \triangleq \lim_{\Delta \to \infty} \frac{\mathcal{F}_{\mathcal{A}}(N+\Delta, S_2)/(N+\Delta)}{\mathcal{F}_{\mathcal{A}}(N, S_1)/N} \geq O(1). \tag{2}$$

In other words, this means that we consider an architecture scalable if the network-wide performance attribute, scaled by the number of nodes involved, does not decrease with an increasing number of nodes and a non decreasing problem space. Although this definition does not include the cost of maintaining or building the architecture/topology, it can easily be included by means of an additional normalizing quotient in (2).

2.2 Optimality

We are also interested in the optimality of a given architecture, i.e. network topology with associated communication protocols. With this in mind, an optimal architecture $\hat{\mathcal{A}}$ w.r.t. attribute $\mathcal{F}$ is defined as the one which, over all possible architectures $\mathcal{A}$, maximizes η. Note that although this definition is intuitive, it is often difficult to prove optimality over all possible architectures $\mathcal{A}$. In the sequel, however, we will consider the optimum architecture over a sub-set of all possible architectures; for instance, we will consider only flat architectures or only hierarchical ones.

3 Scaling Laws

The first question we pose is when a network has to be considered large. To exemplify this problem, let us presuppose systems with and without internal conflicts [4]. For instance, two systems without conflicts are

1. our circle of true friends, comprising a small number of elements; and
2. the soldiers of an ant colony, comprising a large number of elements.

On the other hand, two systems with conflicts, frictions and competition are, for example,

1. a few children left on their own, comprising a small number of elements; and,
2. a state without government, comprising a large number of elements.

As such, 'large' is hence not about size [4]. It is rather about managing existing and emerging conflicts, and hence the amount of overheads needed to facilitate (fair) communication. This overhead is well reflected in the efficiency η, which needs to be maximized for a given attribute $\mathcal{F}$. If the attribute is e.g. throughput, then a flat topology is unlikely to be the optimum architecture, whereas a hierarchical might be a good choice. Using different scaling laws, we will use different attributes to judge upon the scalability of considered architectures.

3.1 Kumar & Gupta's Throughput Scaling Law

This milestone contribution [9] quantifies the theoretically achievable per-node capacity assuming that every node wishes to communicate with every other node. The architecture is assumed to be flat and hence does not contain any structural elements, such as hierarchies or clusters. They have determined that, assuming random deployment of the nodes in a unit area with N nodes, the per-node capacity scales with $1/\sqrt{N \log N}$ bits/s and the average network capacity

$$\Theta \propto \frac{N}{\sqrt{N \log N}}. \tag{3}$$

To gauge scalability as per (2), one needs to determine architecture, attribute and problem size. According to [9], the architecture $\mathcal{A}$ is flat, i.e. it contains no hierarchical structure, nor clusters, nor any hybrid constructions thereof. The attribute $\mathcal{F}$ is the average network capacity Θ. Finally, with the average network capacity being the 'problem' to be solved in the network, the problem size S is related to the total number

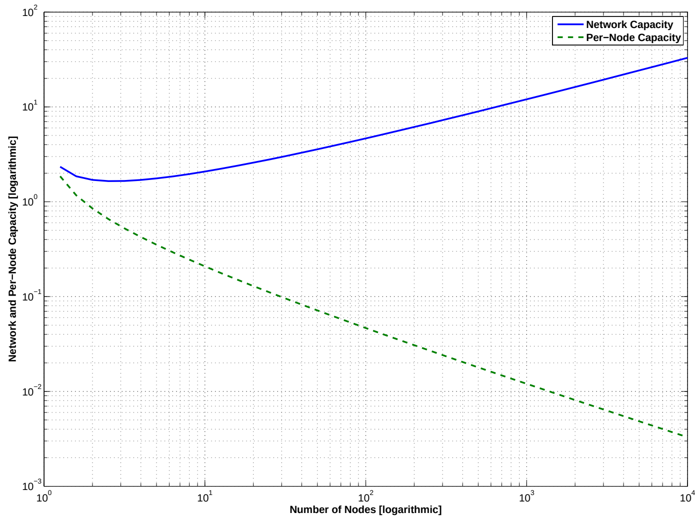

Fig. 2 Kumar & Gupta's increasing network and diminishing per-node capacities.

of nodes in the network N (thus certainly not decreasing with an increasing number of nodes). Inserting (3) into (2), one hence gets

$$\eta = O(1/\sqrt{\Delta \log \Delta}) < O(1), \tag{4}$$

revealing that such architecture is not scalable w.r.t. the network capacity. This seems to be at odds with Figure 2, which shows in double-logarithmic scale the average network capacity increases with an increasing number of nodes. However, this increase is not fast enough to guarantee non-diminishing throughput per node (i.e., each problem in S to be attended to with the same resources). Indeed, as per Figure 2, the throughput per node decreases rapidly with an increasing number of nodes. In other words, no matter what we try, we cannot design a protocol for large networks which is scalable according to (2) w.r.t. the total average network throughput.

Since [9] has proven above bounds to be the result of optimum communication protocols, we concur that the only way to achieve some form of scalability is to invoke architectures different from flat ad hoc. This has recently been considered in [10], in which it has been proven that network capacity can increase linearly (and hence per-node capacity remains constant) for an increasing number of nodes by using cooperative communication architectures; this tendency has already been alluded to in [11].

3.2 Odlyzko & Tilly's Value Scaling Law

Moving away from traditional quantifications of a network by means of throughput, etc., we wish to determine the relative value of large networks. Mainly economically driven, various efforts in the past by e.g. Sarnoff, Reed and Metcalf have been dedicated to establishing the value of a network in dependency of the number of its elements N:

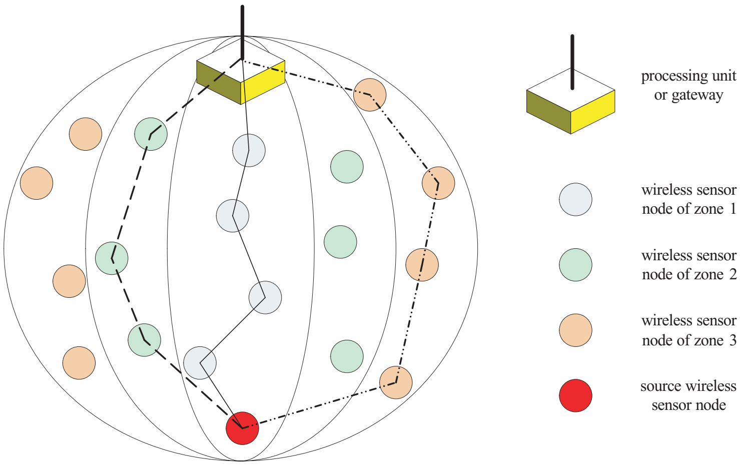

Fig. 3 Value zones as seen from the source node towards the sink, where the value of each zone obeys Zipf's Law.

- Sarnoff's Law [12] quantifies the value of a broadcast network to be proportional to N, which stems from the fact that the N members only communicate with the BS.
- Reed's Law [13] claims that with N members you can form communities in 2^N possible ways; the value hence scales with 2^N.
- Metcalfe's Law [14], unjustifiably blamed for many dot-com crashes, claims that N members can have $N(N-1)$ connections; the value hence scales with N^2.

Value here is clearly a relative notion which could reflect monetary value, importance, etc. The relative character could e.g. allow one to estimate the increase of return in money or importance when increasing the number of members from N to $N + \Delta N$. It does not, however, allow one to gauge the absolute value of a community.

Since a large-scale network – be it a cellular network or a WSN – is not truly of broadcast nature, nor do nodes form all possible communities, nor does every node communicate with every other node, another value scaling law is required to quantify the network's behavior.

To this end, Odlyzko and Tilly have proposed a value scaling which is proportional to $N \log N$ [15]. Their argumentation bases on Zipf's Law [16], an important law in biology and medicine with discrete samples, which states that if one orders a large collection of entities by size or popularity, the entity ranked k-th, will be in value about $1/k$ of the first one. The added value of a node to communicate with the remaining nodes is hence $\sum_{n=1}^{N} 1/n \propto \log N$. This can equally be formulated for continuous values leading to the same result since $\int_{n=1}^{N} 1/n \, dn \propto \log N$. The total value V of the network with N nodes hence scales with

$$V \propto N \log N. \tag{5}$$

Among others, this law has been found to describe accurately the merging and partitioning of companies of unequal size [15]. It can also be used to describe the routing behavior in WSNs, which is exemplified in Figure 3. All nodes, which are or can potentially be used to transmit information from a source towards a sink with

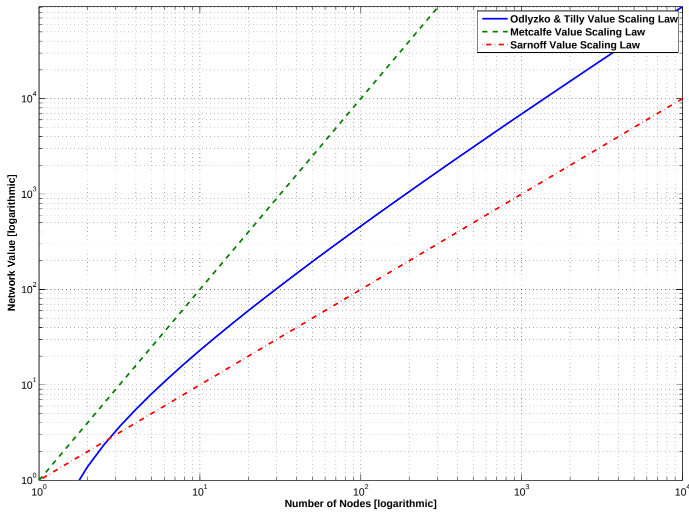

Fig. 4 Odlyzko & Tilly's network value augments faster than Sarnoff's value but slower than Metcalfe's value.

energy E_1, are grouped in value zone 1; the value of E_1 is determined such that $E_{\min} \leq E_1 < 2 \cdot E_{\min}$, where $E_{\min}$ is the minimum needed energy. Equivalently, all nodes which require energy $k \cdot E_{\min} \leq E_k < (k+1) \cdot E_{\min}$ to route the information from source to sink are placed in zone k. The value of the network to the source node with N of such zones is hence proportional to $N \log N$.

Again, to gauge scalability as per (2), one needs to determine architecture, attribute and problem size. In this case, the architecture $\mathcal{A}$ has prioritized nodes, the network attribute $\mathcal{F}$ is its value V and the problem size S is related to the total number of nodes in the network N per unit area (thus certainly not decreasing with an increasing number of nodes). With reference to (2) and (5), the relative efficiency can hence be calculated as

$$\eta = O(\log \Delta) > O(1), \tag{6}$$

revealing that w.r.t. network value the architecture is scalable. Indeed, from Figure 4 depicting Sarnoff's, Odlyzko & Tilly's and Metcalfe's scaling laws, the network value increases rapidly with an increasing number of nodes. In other words, forming communities is favorable to the scaling of the network value. Such communities could be clusters or value-dependent routing zones in WSNs. From Figure 4, it is also interesting to observe that Sarnoff's and Odlyzko & Tilly's value scaling laws intersect for a low number of nodes in the network (where these laws are well applicable). This indicates that broadcasting is only valuable for small communities. With application to WSNs, and without considering energy consumption in the network, this indicates that broadcasting should only be applied to the immediate neighborhood of a broadcasting node.

4 Application of Scaling Laws

The above fundamental scaling laws gave us the following insights:

1. With respect to (4), the network throughput decreases with an increasing amount of nodes due to the increasing amount of required links and hence counteracting scalability.
2. With respect to (6), the network value increases with an increasing amount of nodes due to the increasing amount of facilitated links and hence enabling scalability.

This apparent discrepancy is due to both laws describing two inherently different but dependent attributes of an architecture. Indeed, the value of an architecture cannot be guaranteed if the throughput over the required links cannot be maintained. Subsequently, we hence aim at exploiting and trading this dependency to find architectures optimum under given assumptions.

Whilst not proven to be the optimum architectural solution $\hat{\mathcal{A}}$, the question which naturally arises in this context is whether clusters and hierarchies will improve the scalability of the architecture.

As described before, clusters in form of cells and hierarchies incorporating tiers of mobile stations, tiers of node Bs, tiers of radio network controllers, etc., have been used with success in cellular networks. Clustering has also been introduced as a means of structuring a wireless multihop network, such as WSNs.

It is commonly assumed that using such a hierarchical architecture yields better results than using a flat topology where all nodes have the same role. In particular, it is assumed that clusters can reduce the volume (but not contents!) of inter-node communication, and hence increase the network's lifetime.

To our knowledge, solely [17] has formally studied these aspects for WSNs only with focus on energy consumption. The authors have shown that clustered architectures out-perform non-clustered ones in a selected number of cases. In particular, the WSN's energy consumption is proven to be reduced only when data aggregation is performed by the cluster-heads.

Subsequently, we will hence examine a few selected clustering approaches and quantify their scalability w.r.t. some important attributes, such as value and throughput.

4.1 Value of Clustered Network Obeying Odlyzko and Tilly's Law

Based on Odlyzko and Tilly's value scaling law, we introduce a normalized network value V', which we define as the ratio between the value given in (5) and the number of links per unit area needed to support such connected community. This definition hence incorporates the required links into the value of the spanned network.

For an unclustered network, we can calculate the normalized network value V' as

$$V' = \frac{N \log N}{N \log N} = 1. \tag{7}$$

For a clustered network, we assume C clusters and hence $M = N/C$ nodes per cluster. Assuming that the value of the nodes within a cluster as well as the cluster heads obeys Zipf's Law, the value per cluster is $M \log M$ and − with about $C \log C$ clusters being active/valuable − the value of the clustered network is $C \log C \cdot M \log M$. The average number of links needed to maintain all nodes and clusters at any time is

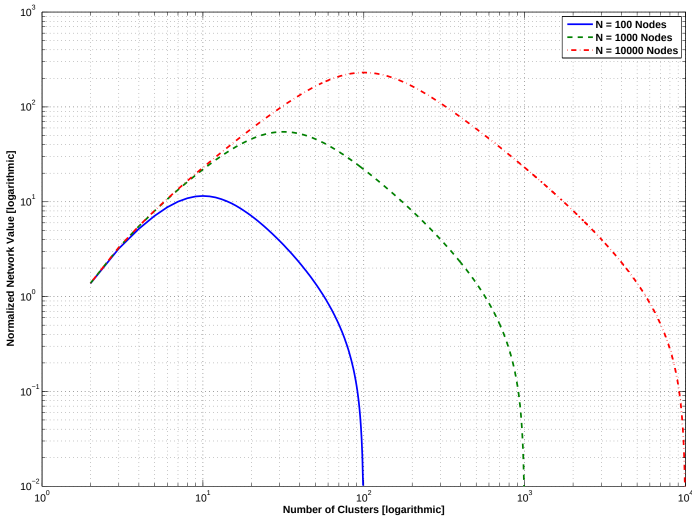

Fig. 5 Relative network value (8) when using clusters.

$C \log C + M \log M$. Note that we have deliberately not chosen $C \log C + CM \log M$ as we wish to normalize w.r.t. the spectral utilization and not energy cost per link (see subsequent sections on the throughput calculation in the context of clustered networks). We can hence calculate the normalized network value V' as

$$V' = \frac{C \log C \cdot M \log M}{C \log C + M \log M}. \tag{8}$$

The relative network value for different cluster sizes is depicted in Figure 5 with $N = \{100, 1000, 10000\}$ nodes. Clearly, clustering increases the normalized network value. We observe that a different number of nodes yields a different optimum number of clusters and a different normalized network value. The optimum cluster size C can be derived from (8) using Lagrangian optimization yielding $C = \sqrt{N}$, which constitutes the optimum architecture $\hat{\mathcal{A}}$ from the reduced set of possible clustered architectures.

With reference to (2), the architecture $\mathcal{A}$ is clustered, the attribute $\mathcal{F}$ is V' and the problem size S is related to the total number of nodes in the network N (thus certainly not decreasing with an increasing number of nodes). For an optimal cluster size of $C = \sqrt{N}$, the relative efficiency can hence be calculated as

$$\eta = O(\log \sqrt{\Delta} / \sqrt{\Delta}) < O(1), \tag{9}$$

revealing that w.r.t. the normalized network value the architecture is not scalable. However, if compared to an unclustered scheme which exhibits a relative efficiency of $\eta = O(1/\Delta)$, the clustered approach clearly exhibits a better scalability. This is corroborated by Figure 6. The scalability properties of the schemes analyzed so far are summarized in Table 1.

System Metric	Upper Bound on Efficiency η	Scalability
$\Theta = \frac{\sqrt{N}}{\sqrt{\log N}}$	$O\left(\frac{1}{\sqrt{N \log N}}\right)$	no
$V = N \log N$	$O(\log N)$	yes
$V' = \frac{\sqrt{N} \log \sqrt{N} \cdot \sqrt{N} \log \sqrt{N}}{\sqrt{N} \log \sqrt{N} + \sqrt{N} \log \sqrt{N}}$	$O\left(\frac{\log \sqrt{N}}{\sqrt{N}}\right)$	no

Table 1 Comparing the scalability behavior with increasing members N.

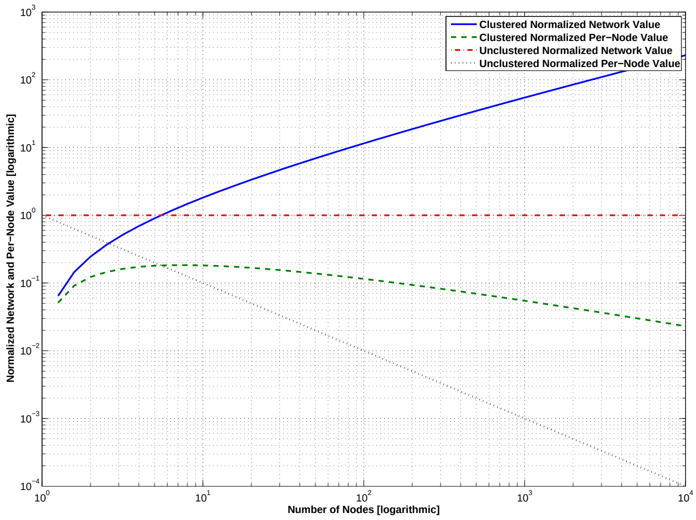

Fig. 6 Relative clustered and unclustered network values; beyond 5 nodes, clustering is always beneficial.

4.2 Throughput of Clustered Network with Fully Meshed Cluster-Heads

We now wish to shed light onto the requirements of the architecture's data pipes which is, for instance, applicable to cellular systems with meshed network controllers or WSNs with meshed cluster heads. We hence assume that each node communicates only with its respective cluster head and all cluster heads communicate among each other, leading to a 2-tier hierarchy with N total nodes, C clusters with one cluster head and $M = N/C - 1 \approx N/C$ nodes per cluster. This 2-tier hierarchy requires two communication phases. First, all nodes communicate with their respective cluster heads; and second, all cluster heads communicate among each other. We assume first all data pipes to have equal rates and then extend this to unequal rates.

Assuming equal data pipes between all nodes, there are $T_1 = M$ time slots in the first phase to transmit $c \cdot N$ bits, where c is a constant and, without loss of generality, is assumed to be one; we also assume that clusters do not interfere with each other in the first phase. In the second phase, one needs $T_2 = M \cdot C \cdot (C - 1)$ time slots to transmit these bits to every cluster head.

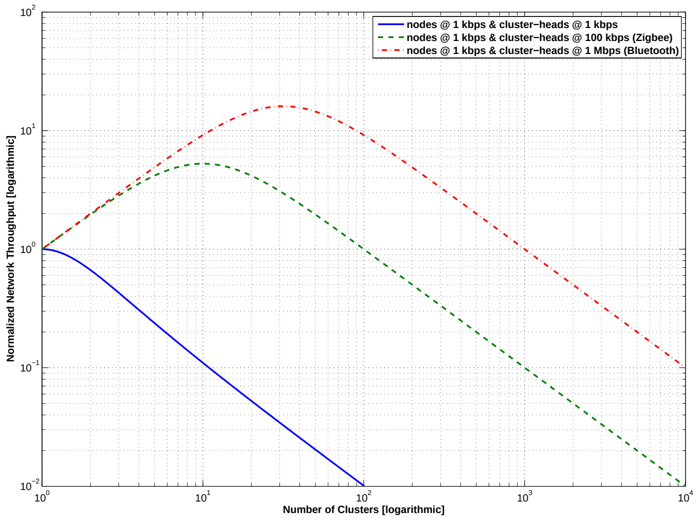

Fig. 7 Normalized network throughput for different data pipes assuming fully meshed cluster heads.

Remembering that no new information is injected in the second phase, the normalized throughput is

$$\begin{aligned}\Theta &= \frac{N}{T_1 + T_2} \\ &= \frac{N}{M + M \cdot C \cdot (C-1)}. \end{aligned} \tag{10}$$

For unequal data pipes, let us assume the cluster-heads' pipes to be α times stronger than the data pipes between nodes towards the cluster heads; alternatively, this can also be achieved if the cluster heads perform data aggregation [19], leading to α-times less data volume to be forwarded. Therefore, in the first phase, there are again $T_1 = M$ time slots to transmit a total of N bits and, in the second phase, there are $T_2 = M \cdot C \cdot (C-1)/\alpha$ time slots to transmit these N bits. The throughput is hence

$$\Theta = \frac{N}{M + M \cdot C \cdot (C-1)/\alpha}. \tag{11}$$

Assuming a WSN, the normalized network throughput for different cluster sizes is depicted in Figure 7 with $N = 10000$ nodes. Clearly, clustering increases the normalized network throughput only if the data pipes among the cluster heads are stronger or data aggregation is performed to decimate the information shared among cluster heads. For instance, if we assume the cluster heads' data pipes to be 1000 times stronger, then an optimal cluster number is approximately 32. The exact optimum number of clusters can be calculated using Lagrangian optimization to be equal to $\sqrt{\alpha}$, i.e. it is independent of the number of nodes in the network. If not all cluster heads communicated, as in this example, then the optimal cluster number would be larger. Note further that in both clustered and un-clustered cases the relative efficiency is given by $\eta = O(1/\Delta)$, revealing that w.r.t. the normalized network throughput the architecture is not scalable.

Above quantification of the normalized network throughput and value of a large network hence stipulate the use of clustered approaches. This is corroborated by real-world roll-outs, all of which use hierarchical and/or clustered network topologies with stronger data pipes between cluster heads. For example, the currently functioning meter reading application of Coronis uses a hierarchical approach [20] and so does Intel's WSN [21].

4.3 Throughput of One-Hop Clustered Network

In this subsection, we assume that all nodes can communicate with their cluster heads over a single hop (1-hop clusters), and that also all cluster heads can reach the sink in a single hop. We assume again that all nodes have N bits to transmit towards the sink node. We compare the throughput when using a clustered structure close to LEACH [22] and direct communication.

For the clustered approach, we assume again the data pipes from cluster heads towards sink to be α-times stronger than between sensor nodes and cluster heads. Therefore, in the first phase, there are again $T_1 = M$ time slots to transmit N bits and, in the second phase, there are $T_2 = M \cdot C/\alpha$ time slots to transmit these N bits. The throughput is hence

$$\begin{aligned} \Theta &= \frac{N}{M + M \cdot C/\alpha} \\ &= \frac{\alpha \cdot C}{\alpha + C}, \end{aligned} \tag{12}$$

which is depicted in Figure 8. It can clearly be seen that clustering improves performance if the inter-cluster data pipes are stronger than the intra-cluster pipes. Also, note that with the number of clusters $C \to \infty$, the normalized throughput $\Theta \to \alpha$. Finally, comparing with Figure 7, it can be observed that the absolute normalized throughput values are larger for the one-hop case compared to the fully meshed one.

For the flat topology, for the sake of a fair comparison with the clustered approach, we consider C nodes having a bandwidth α times stronger than other nodes; we call them super-nodes. These super-nodes would play the role of cluster heads in a clustered structure. In this case, only one phase is needed which is composed of $T_1 = (N - C) + C/\alpha$ time slots. The normalized throughput of the flat topology is hence

$$\begin{aligned} \Theta' &= \frac{N}{(N - C) + C/\alpha} \\ &= \frac{\alpha}{\alpha + C/N \cdot (1 - \alpha)} \end{aligned} \tag{13}$$

This allows us to establish the conditions under which clustering is worthy by obtaining the difference $\Delta\Theta$ between (12) and (13). It can easily be shown that clustering improves performance if the number of nodes N and super-nodes C relate as follows:

$$N > \frac{C^2 \cdot (1 - \alpha)}{C \cdot (1 - \alpha) + \alpha}. \tag{14}$$

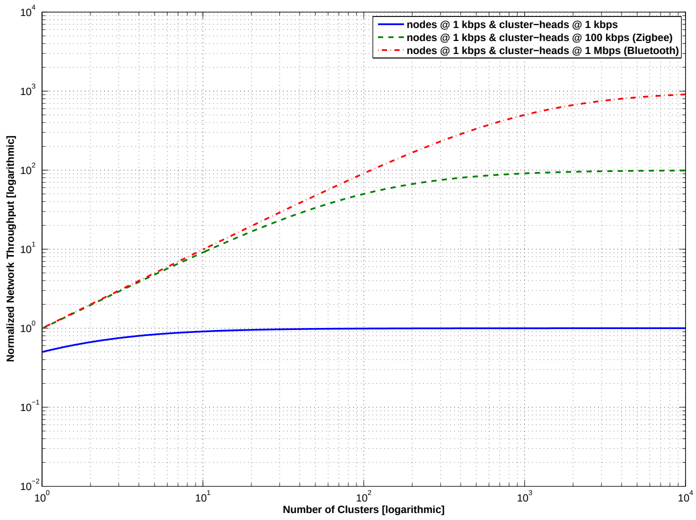

Fig. 8 Normalized network throughput for different data pipes assuming a one-hop clustered network.

This is plotted in Figure 9, which shows that for a large number of super-nodes C the condition of clustering being beneficial is simply $C/N < 1$ and for a small number of super-nodes the number of nodes N has to be large. Note that these results are independent of the node density as long as the assumption on the one-hop reachability can be guaranteed.

4.4 Throughput of Multi-Hop Clustered Network

In this final subsection, we assume that all nodes can communicate with their cluster heads over a single hop (1-hop clusters), but that all cluster heads can only reach the sink via multiple hops. To facilitate subsequent analysis, we assume the topology model of [9], consisting of a unit area circular domain with the sink in the center; this is illustrated in Figure 10. Such a deployment scenario is envisaged for real-world WSN roll-outs, where the transmission power and hence also communication radius of sensor nodes are severely limited due to the nodes' stringent energy constraints.

To analyze this topology, we need to know the average number of hops from source to sink. To this end, the average distance from any point of that domain to the sink can be calculated as $\bar{L} = \int_0^{\frac{1}{\sqrt{\pi}}} r \cdot 2\pi \cdot r dr = \frac{2}{3\sqrt{\pi}}$. As per Figure 10, nodes are uniformly scattered in the unit area domain and, to consume less energy, keep their transmission power (thus communication range) as low as possible. Each node hence covers $\frac{1}{N}$ area, which is equivalent to a circle of diameter $\frac{1}{\sqrt{\pi N}}$. Two neighbor nodes are thus at a distance of $\bar{r} = \frac{2}{\sqrt{\pi N}}$ and the average number of hops $\bar{h} = \frac{\bar{L}}{\bar{r}}$ between a node and the sink can finally be calculated as $\bar{h} = \frac{\sqrt{N}}{3}$.

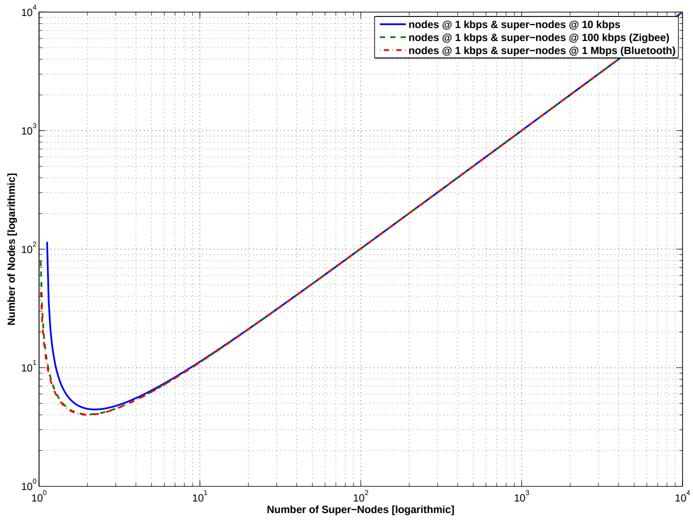

Fig. 9 Number of nodes versus number of super-nodes, where configurations above the lines yield performance gains.

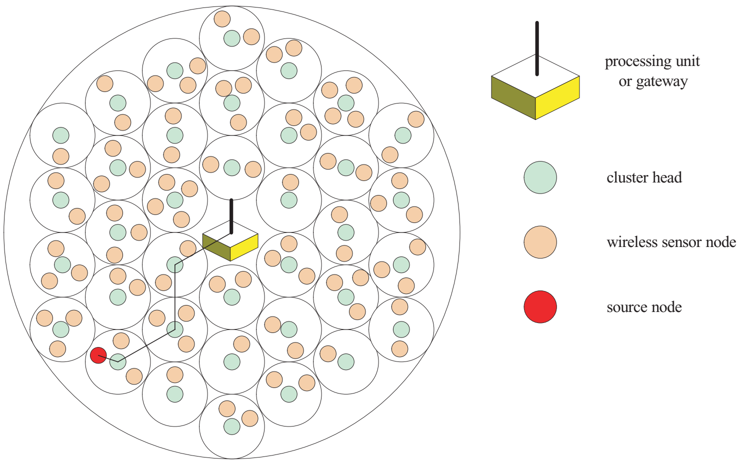

Fig. 10 Nodes and clusters are uniformly scattered in the circular domain.

To calculate the normalized throughput, we assume again the inter-cluster data pipes to be α-times stronger than the intra-cluster pipes. In the first phase, there are again $T_1 = M$ time slots to transmit the N bits. In the second phase, each of the C cluster heads sends the collected M bits to the sink over a path of average length of $\frac{\sqrt{C}}{3}$ hops, where – according to [9] – each cluster head has an available bandwidth of $\frac{\alpha}{\sqrt{C \log C}}$. As a consequence, this second phase takes $T_2 = \frac{MC}{3\alpha}\sqrt{\log C}$ time slots. The

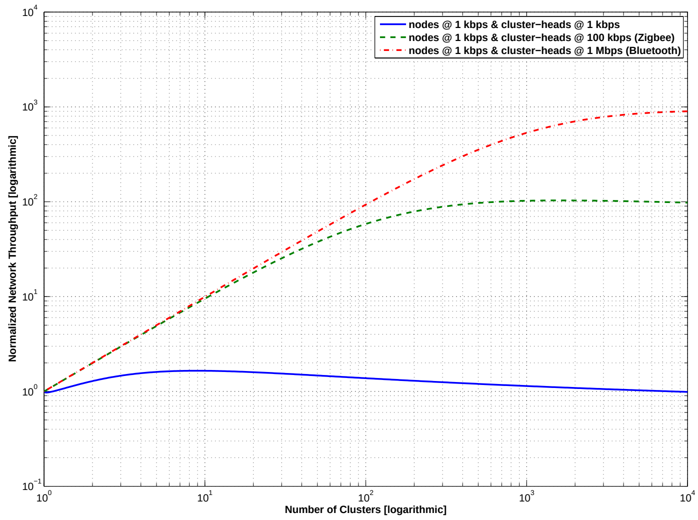

Fig. 11 Normalized network throughput for different data pipes assuming a multi-hop clustered network.

normalized network throughput can hence finally be written as

$$\Theta = \frac{3 \cdot C \cdot \alpha}{3 \cdot \alpha + C \cdot \sqrt{\log C}}, \tag{15}$$

which is depicted in Figure 11. The condition for the maximum cannot be derived in closed form; however, for α not too large, the maximum is approximately achieved at $C \approx 10\alpha$.

To obtain the normalized throughput for a flat topology, we will following the same reasoning as in the previous section. To this end, we assume again the existence of C super-nodes possessing $\alpha-$times stronger data pipes. However, for a fairly low density of these super-nodes, they can only connect with nodes of low bandwidth rendering the higher bandwidth superfluous. With this assumption in mind, the average number of slots for the single communication phase is simply $T_1 = \frac{N}{3}\sqrt{\log N}$. The normalized throughput of the flat topology is hence:

$$\Theta' = \frac{3}{\sqrt{\log N}}. \tag{16}$$

This allows us to establish the conditions under which clustering is worthy by obtaining the difference $\Delta\Theta$ between (15) and (16). It can easily be shown that clustering improves performance if the number of nodes N and super-nodes C relate as follows:

$$N > \exp\left[\left(\frac{3}{C} + \frac{\sqrt{\log C}}{\alpha}\right)^2\right]. \tag{17}$$

5 Concluding Remarks

The aim of this paper was to expose some crucial issues related to the scalability and design of large wireless networks. Using some well established scaling laws from communications, i.e. Kumar & Gupta's throughput scaling law, and economics, i.e. Odlyzko & Tilly's value scaling law (which is based on Zipf's law), we have established that large scale networks generally do not scale w.r.t. some key attributes with an increasing number of nodes.

To quantify scalability, we have introduced the notion of architectural efficiency and defined an architecture to be scalable if this efficiency is at least of the order of $O(1)$. This definition has facilitated a comparison between the scalability of unclustered and clustered architectures.

Whilst both unclustered and clustered architecture was shown not to be scalable, a clustered approach $-$ based on some given assumptions $-$ has shown to exhibited a better scalability than its unclustered counterpart. We have also shown that, if clustering is used, the data pipes between the cluster heads need either to be stronger or data aggregation needs to be performed at the cluster-heads. Such clustered architectures can be built using self-organizing and self-healing algorithms, such as introduced in [23].

These results are clearly indicative only, where different assumptions on attributes $\mathcal{F}$, inclusion of energy consumption, choice of hierarchy, choice of data flows (e.g., directed towards WSN sink), etc., will yield different absolute results. Nonetheless, the results expose tendencies which, so we hope, are of use for large-scale system designers and hence for emerging and future real-world applications, such as data collection, remote control solutions, wireless telemetry, automatic monitoring, metering solutions and smart environments such as homes, hospitals, and buildings of all kinds.

References

1. M. Dohler, T. Watteyne, D. Barthel, F. Valois, J. Lu "Kumar's, Zipf's and Other Laws: How to Structure an Optimum Large-Scale Wireless (Sensor) Network?," *European Wireless 2007,* Conference CD-ROM, Paris, France, April 2007.
2. RNRT ARESA Project, ANR-05-RNRT-01703; see also http://aresa-project.insa-lyon.fr/.
3. M. Dohler, S. Aubert, F. Valois, O. Bezet, F. Maraninchi, L. Mounier, F. Paugnat, *Definition of needs and usage scenarios,* Deliverable D1.2, RNRT ARESA Project, ANR-05-RNRT-01703.
4. M. Dohler, D. Barthel, S. Aubert, C. Dugas, F. Maraninchi, M. Laurent, A. Buhrig, F. Paugnat, M. Renaudin, A. Duda, M. Heusse, F. Valois, "The ARESA Project: Facilitating Research, Development and Commercialization of WSNs," *IEEE SECON 2007,* Conference CD-ROM, San Diego, USA, June 2007.
5. Private communication with various industrials at *International Conference on WSNs,* November 2006, Paris, France.
6. Mark D. Hill, "What is scalability?," *ACM SIGARCH Computer Architecture News*, December 1990, Volume 18 Issue 4, pages 18-21.
7. J. Habetha, J. Wiegert, "Network capacity optimisation, Part 2: Multihop ad hoc radio networks," *Proceedings of 10th Symposium on Signal Theory,* Aachen, September 2001, pp. 133-140.
8. C.A. Santivanez, B. Mc Donald, I. Stavrakakis, R. Ramanatha, "On the scalability of ad hoc routing protocols," *IEEE INFOCOM,* NY, USA, June 2002, pp. 1688-1697, vol. 3.
9. P. Gupta, P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory,* vol. 46, pp. 388-404, 2000.
10. A. Ozgur, O. Leveque, D. Tse, "Hierarchical Cooperation Achieves Optimal Capacity Scaling in Ad Hoc Networks," to appear in the *IEEE Transactions on Information Theory*, December 2007.

11. M. Dohler, *Virtual Antenna Arrays,* PhD Thesis, King's College London, London, UK, 2003.
12. K.M. Bilby, *The General: David Sarnoff and the Rise of the Communications Industry,* New York: Harper & Row, 1986.
13. D.P. Reed, "Weapon of math destruction: A simple formula explains why the Internet is wreaking havoc on business models," *Context Magazine,* Spring 1999.
14. G. Gilder, "Metcalfe's Law and Legacy," *Forbes ASAP,* 1993.
15. B. Briscoe, A. Odlyzko, B. Tilly, "Metcalfe's Law is Wrong," *IEEE Spectrum Magazine,* July 2006.
16. G.K. Zipf, "Some determinants of the circulation of information," *Amer. J. Psychology,* vol 59, 1946, pp. 401-421.
17. N. Vlajic, D. Xia, "Wireless sensor networks: To cluster or not to cluster?," *International Symposium on a World of Wireless, Mobile and Multimedia Networks (WoWMoM06)*, 2006, pp. 258-268.
18. W. B. Heinzelman, A. P. Chandrakasan, H. Balakrishnan, "An application-specific protocol architecture for wireless microsensor networks, *IEEE Transactions on Wireless Communications,* vol. 1, no. 4, pp. 660-670, 2002.
19. H. Chen, H. Mineno, T. Mizuno, "A Meta-Data-Based Data Aggregation Scheme in Clustering Wireless Sensor Networks," *Mobile Data Management'06*, May 2006, pp. 154-154.
20. C. Dugas, "Configuring and managing a large-scale monitoring network: solving real world challenges for ultra-lowpowered and long-range wireless mesh networks," *Int. J. Network Mgmt,* 2005, vol 15, pp. 269-282.
21. Intel Research; see also www.intel.com/research.
22. W. B. Heinzelman, A. P. Chandrakasan, and H. Balakrishnan, "An application-specific protocol architecture for wireless microsensor networks, *IEEE Transactions on Wireless Communications,* vol. 1, no. 4, pp. 660-670, 2002.
23. J.-L. Lu, F. Valois, D. Barthel, "Low-Energy Self-Organization Scheme for Wireless Ad Hoc Sensor Network," *IEEE WONS,* January 2007, pp. 138-145.